Uniformly accelerated particles and gravitational waves: The Unruh effect and zero-Rindler-energy modes

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Abstract

Acceleration is one of the mechanisms that produces radiation, meaning gravitational waves should be emitted by accelerating masses. Here we describe the gravitational radiation coming from a uniformly accelerated particle in both the classical and quantum pictures, using Unruh modes to decompose the fields. This allows us to study the role of the Unruh effect in both descriptions, and find, like in the electromagnetic and scalar cases, that zero-Rindler-energy modes (and their associated gravitons) have a fundamental role in the build-up of the radiation content seen by inertial observers.

Introduction

Since the discovery of the relationship between acceleration and radiation, many physicists have contributed to improve our understanding of this connection. Classically we know that radiation is not a covariant concept: the state of motion of the observer determines how radiation will be perceived [1, and references within]. Our grasp on the quantum aspects of this correspondence are based on the finding that an accelerated observer will see the Minkowski vacuum as a bath of particles at a temperature proportional to its proper acceleration [2], what we now call the Unruh effect. Other developments on Quantum Field Theory in Curved Spacetimes show that for a uniformly accelerated charge, the excitation rates reported by an inertial observer agree with those of an observer co-accelerated with the charge, if and only if the thermal bath is taken into account; also, surprisingly, the photons appearing in this set-up have zero energy in the Rindler frame [3]. This idea was exploited recently to propose an experimental setup where the quantum radiation emitted by an accelerated charge thoroughly reproduces Larmor radiation [4]. The authors have explored the role of zero-Rindler-energy photons in this description and the interplay between the Unruh effect and classical Larmor radiation in both scalar [5] and vector [6] electrodynamics, and now have extended the methods used in these studies to the linearized gravitational field $h_{ab} = \delta g_{ab}$.

Particles accelerated for a finite amount of time

The trajectory of a uniformly accelerated particle of mass m in Minkowski spacetime is described by an hyperbola, which is not a geodesic of this spacetime. Therefore the dynamics of the particle are governed by a modification of the geodesic equation

$$\ddot{\chi}^a + \Gamma^a{}_{bc}\dot{\chi}^b\dot{\chi}^c = F^a(\chi),$$

where $\chi^a(\tau)$ is the trajectory, and $F^b\partial_b = a^2(t\partial_t + z\partial_z) = a\partial_\xi$ is a vector field representing the external agent responsible for the acceleration. This corresponds to the Euler-Lagrange equation of an action principle, that, after introducing a compactification parameter L and half the time of acceleration T, yields the energy-momentum tensor for the particle:

$$T_L^{ab}(x) = \frac{m}{\sqrt{-g(x)}}\theta(L-|t|)\int_{-\infty}^{\infty}\left[\frac{d\chi^a}{d\tau}\frac{d\chi^b}{d\tau} + \frac{\theta(T-|\tau|)}{a^2}F^aF^b\right]\delta^4(x-\chi(\tau))\,d\tau.$$

The physical setup is recovered by making $L \to \infty$, and we will give special interest to the case $T \to \infty$. The trajectory can be visualized in the figure below.

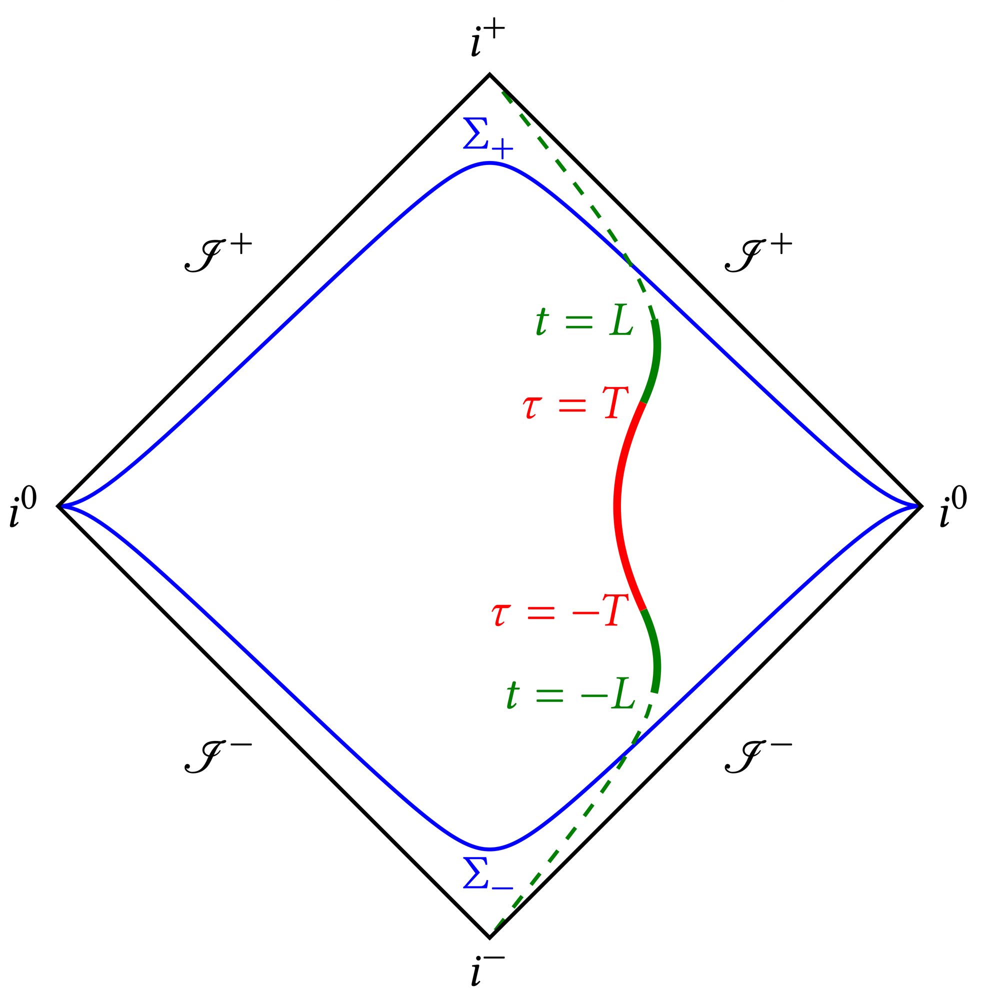

Figure: Conformal diagram of the motion of the particle. The accelerated part of the motion is in red and constrained to $|\tau| \le T$, while the inertial parts are in green. The compactification parameter limits the support of the trajectory to $|t| \le L$ and the limit $L \to \infty$ recovers the physical trajectory (dotted sectors). The asymptotic past and future Cauchy hyper-surfaces are in blue, and do not intersect with the compactified trajectory.

Traceless and transverse tensor Unruh modes

Taking advantage of the planar symmetry of Rindler and Minkowski spacetime we can apply the procedure outlined by Kodama et al [7] and define both left and right Rindler modes, alongside with plane wave modes. A simple analysis shows that these Rindler modes satisfy the same Bogoliubov transformations than the scalar counterparts of the modes. From this we can define a normalised, positive energy, and complete set of Unruh modes; these are classified as the *vector sector*

$$W_{ab}^{(\sigma,v,\omega\mathbf{k}_\perp)} = -\frac{i\sqrt{16\pi G}}{k_\perp^2}\left(\begin{array}{c|c} 0 & \epsilon_{\alpha\beta}\varepsilon_{jl}\nabla^\beta\nabla^l w^\sigma_{\omega\mathbf{k}_\perp} \\ \hline \epsilon_{\alpha\beta}\varepsilon_{jl}\nabla^\beta\nabla^l w^\sigma_{\omega\mathbf{k}_\perp} & 0 \end{array}\right),$$

and the *scalar sector*

$$W_{ab}^{(\sigma,s,\omega\mathbf{k}_\perp)} = \frac{\sqrt{16\pi G}}{k_\perp^2}\left(\begin{array}{c|c} 2\nabla_\alpha\nabla_\beta w^\sigma_{\omega\mathbf{k}_\perp} - k_\perp^2 g_{\alpha\beta}w^\sigma_{\omega\mathbf{k}_\perp} & \nabla_\alpha\nabla_j w^\sigma_{\omega\mathbf{k}_\perp} \\ \hline \nabla_\alpha\nabla_j w^\sigma_{\omega\mathbf{k}_\perp} & 2\nabla_i\nabla_j w^\sigma_{\omega\mathbf{k}_\perp} + k_\perp^2 g_{ij}w^\sigma_{\omega\mathbf{k}_\perp} \end{array}\right),$$

where we have used the definition of the scalar Unruh modes

$$w^\sigma_{\omega\mathbf{k}_\perp}(x) = \frac{e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}}{4\pi^2\sqrt{2a}}\int_{-\infty}^{\infty}\exp\left[i\left(\frac{(-1)^\sigma\vartheta\omega}{a} + k_\perp(z\sinh\vartheta - t\cosh\vartheta)\right)\right]d\vartheta, \qquad \sigma = 1,2.$$

There have been previous reports of gravitational Rindler modes [8], but these were derived in the Regge-Wheeler gauge and in a different fashion from ours.

Classical expansion of field in the asymptotic future

We can describe the gravitational perturbation in the asymptotic future (Σ_+) using the retarded field $h_{ab} = RT_{ab}$, obtained using the stress-energy tensor and Green's function methods. This gravitational perturbation can be expanded in terms of our TT Unruh modes, where we find only the scalar sector couples with the energy-momentum tensor. In the special case where the acceleration time is infinite, we computed the coefficients to be

$$\langle W^{(\sigma,v,\omega\mathbf{k}_\perp)}, RT\rangle = 0, \qquad \langle W^{(\sigma,s,\omega\mathbf{k}_\perp)}, RT\rangle = im\sqrt{\frac{8G}{\pi a}}K_2(a^{-1}k_\perp)\,\delta(\omega), \qquad \sigma = 1,2,$$

and therefore the full retarded field is given by

$$RT_{ab}(x) = im\sqrt{\frac{8G}{\pi a}}\iint_{\mathbb{R}^2}K_2(a^{-1}k_\perp)\left(W_{ab}^{(2,s,0\mathbf{k}_\perp)}(x) - \overline{W_{ab}^{(2,s,0\mathbf{k}_\perp)}(x)}\right)d^2\mathbf{k}_\perp,$$

We can see explicitly from the expression above that only zero-Rindler-energy modes ($\omega = 0$) contribute to build this expression.

Quantum expansion of field in the asymptotic past and future

If we promote the gravitational perturbation and the associated generalized momenta to operators and impose the canonical equal-time commutation relation between them, we can see that an observer in the asymptotic past will have its own concept of particle, independent of what the future observes defines as such. This is realized by considering two distinct but equivalent forms of writing the quantised gravitational perturbation

$$\hat{h}_{ab}(x) = \hat{h}^{out}_{ab}(x) + AT^L_{ab}(x)\hat{\mathbb{1}} = \hat{h}^{in}_{ab}(x) + RT^L_{ab}(x)\hat{\mathbb{1}},$$

where $\hat{h}^{out}_{ab}$ and $\hat{h}^{in}_{ab}$ are the free homogeneous fields as seen by observers in the future and past, respectively. Each of these has its own vacua, $|0^M_{out}\rangle$ and $|0^M_{in}\rangle$, and these can be connected using the S matrix: $|0^M_{in}\rangle = \hat{S}|0^M_{out}\rangle$, which, if we expand the out-field using tensor Unruh modes, depends on the coefficients found on the classical expansion, and for the case of infinite acceleration time is given by

$$|0^M_{in}\rangle = \bigotimes_{\mathbf{k}_\perp\in\mathbb{R}^2}\exp\left[-\frac{2m^2G}{\pi^2 a}T_{tot}\left[K_2(a^{-1}k_\perp)\right]^2\right]\exp\left[im\sqrt{\frac{8G}{\pi a}}K_2(a^{-1}k_\perp)\,\hat{a}^\dagger_{out}(W^{(2,s,0\mathbf{k}_\perp)})\right]|0^M_{out}\rangle,$$

a superposition of zero-Rindler-energy gravitons. Moreover, the S matrix can be used to show the in-vacuum is a multimode coherent state of the out-field, i.e.,

$$\hat{a}_{out}(\overline{W^{(\sigma,s,\omega\mathbf{k}_\perp)}})|0^M_{in}\rangle = \langle W^{(\sigma,s,\omega\mathbf{k}_\perp)}, RT_L\rangle|0^M_{in}\rangle,$$

and this allows us to show the expectation value of the out field when the state is prepared as the vacuum in the past is simply the retarded field:

$$\langle 0^M_{in}|\hat{h}^{out}_{ab}|0^M_{in}\rangle = RT^L_{ab},$$

therefore all quantum observables will derived from this field, like the gravitational stress-energy tensor, will average out to their classical counterparts.

Summary

- Extended definition of Unruh modes to spin 2.
- Field decomposed using tensor Unruh modes.
- For infinite acceleration time, only zero-Rindler-energy modes contribute to the field.
- The future vacuum is a multimode coherent state.
- The expectation value of the field in the future corresponds with its classical counterpart.
- For infinite acceleration time, the future observer sees the past vacuum as constructed by past zero-Rindler-energy gravitons only.

Ongoing research

We are focusing our efforts into analysing how the initial state of the field influences the response from UdW detectors, using our definition of gravitational Unruh modes.

References

[1] D. G. Boulware, *Ann. Phys. (N.Y.)* **124**, 169 (1980).
[2] W. G. Unruh, *Phys. Rev. D* **14**, 870 (1976).
[3] A. Higuchi, G. E. A. Matsas, and D. Sudarsky, *Phys. Rev. D* **46**, 3450 (1992).
[4] G. Cozzella et al., *Phys. Rev. Lett.* **118**, 161102 (2017).
[5] A. G. S. Landulfo, S. A. Fulling, and G. E. A. Matsas, *Phys. Rev. D* **100**, 045020 (2019).
[6] F. Portales-Oliva and A. G. S. Landulfo, *Phys. Rev. D* **106**, 065002 (2022).
[7] H. Kodama, A. Ishibashi, and O. Seto, *Phys. Rev. D* **62**, 064022 (2000).
[8] Y. Sugiyama, K. Yamamoto, and T. Kobayashi, *Phys. Rev. D* **103**, 083503 (2021).

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