Reconciling classical and quantum electromagnetic radiation through zero-energy Rindler photons

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Abstract

Recent developments in the context of Quantum Field Theory in Curved Spacetimes suggest that the purely quantum Unruh effect is somehow codified within the classical Larmor radiation; however, further insight on this connection is needed. Here we present a study where a point charge is uniformly accelerated for a finite amount of time, to study the resulting radiation field in both the classical and quantum contexts. For this, we extend the definition of Unruh modes to be vector valued, and use these to decompose the field in both regimes. We find that the expectation value of the quantum observables in the asymptotic future coincides with their classical counterparts, provided the state is prepared as the vacuum in the asymptotic past. Moreover, we are able to show that on the limit where the acceleration time is infinite, the radiation content is solely comprised of zero-energy Rindler photons. This clarifies the link between the Unruh effect and Larmor radiation, along with the role played by these zero-energy Rindler photons.

Introduction

The connection between acceleration and radiation was first pointed out by Larmor [1]. The works by Rohrlich [2, 3] and Boulware [4] further explore and shed light on our understanding of this phenomena in the classical regime, by noting that the state of motion of the observer is intrinsically related to its ability to detect radiation coming from a charge. In the quantum context, Unruh [5] showed that an accelerated observer will be bathed by particles at a temperature proportional to its proper acceleration. This effect is closely related with Bremsstrahlung, as the emission and absorption rate of photons seen by both inertial and accelerated observers are congruent only when considering this thermal bath, and these rates are built entirely from zero-energy modes [6]. This connection was strengthened recently, as there have been propositions of experimental settings to show that Larmor radiation codifies the Unruh effect [7]. These ideas have been further explored along the role played by zero-Rindler-energy photons using scalar fields [8]. Here we present a more realistic setting using Maxwell electrodynamics to study the radiation emitted by an accelerated charge. We use Heaviside-Lorentz units for the electromagnetic quantities and $c = \hbar = 1$.

The 4-current associated to the accelerated charge

We use a charge q that is moving inertially in Minkowski spacetime [described using coordinates (t, x, y, z)], then its accelerated with proper acceleration a for a finite amount of proper time $2T$ and then returns to be inertial (see Fig. 1). We name this physical current as $j^a_\infty(x)$. For calculation reasons, we ***compactify the support of the current*** introducing a parameter $L > a^{-1}e^{aT}$:

$$j^a(x) := j^a_\infty(x)\,\theta(L - |t|), \tag{1}$$

such that the physical current is recovered in the limit $L \to \infty$.

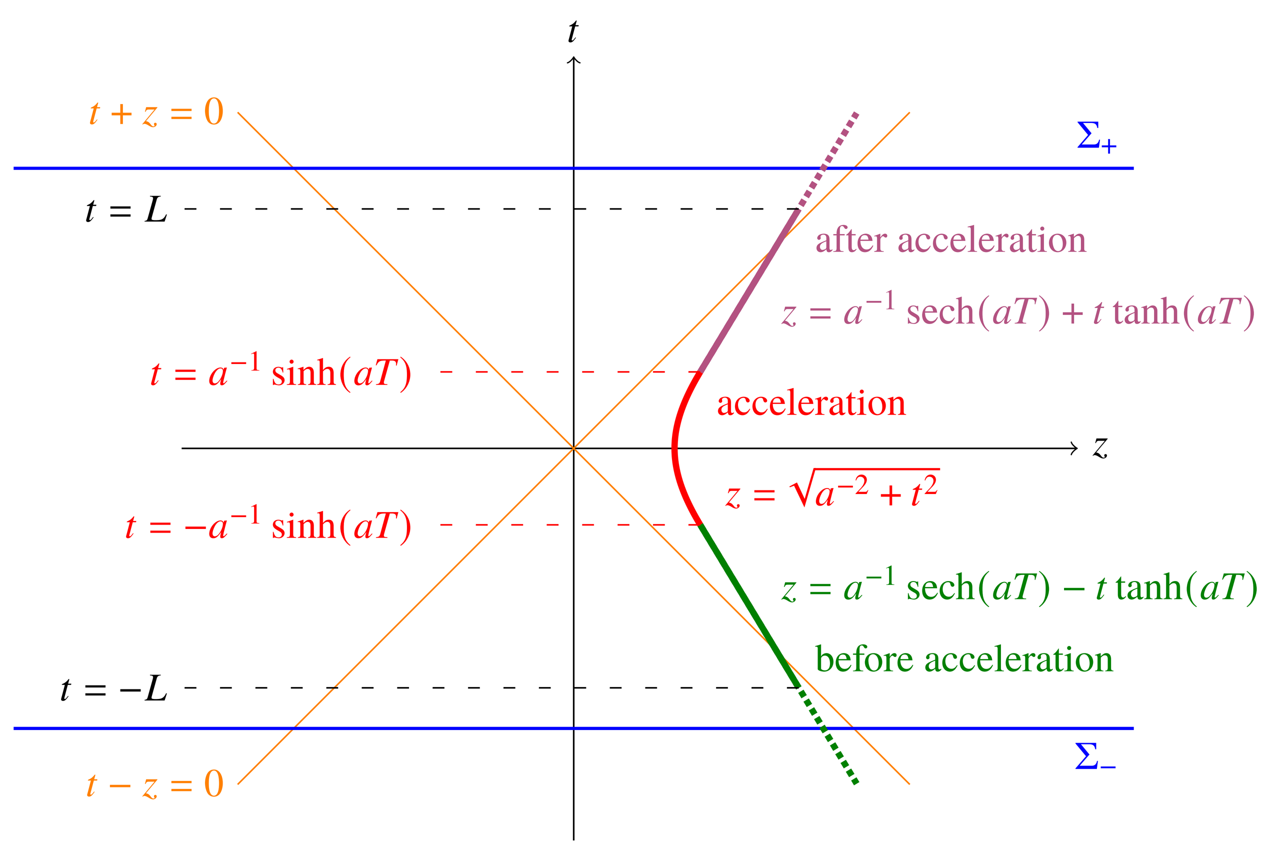

Fig. 1: Support of the current. Inertial parts in green and magenta, while the accelerated part is in red. The compactification of the support corresponds to the solid line; the dotted one to the limit $L \to \infty$. The Cauchy surfaces Σ_- and Σ_+ in blue represent the asymptotic past and future respectively.

Vector Rindler and Unruh modes

On the left and right Rindler wedges ($z < -|t|$ and $z > |t|$ regions respectively), the solutions of the massless Klein-Gordon equation are the left and right Rindler modes $v^L_{\omega,\mathbf{k}_\perp}$ and $v^R_{\omega,\mathbf{k}_\perp}$ respectively. These are positive energy modes on their respective wedges and can be analytically extended to cover the entirety of Minkowski spacetime [9, 10]. From the right modes, we can derive two physical polarizations for the electromagnetic field [6]

$$V^{R(1)}_{\omega\mathbf{k}_\perp\,b} = \frac{1}{k_\perp}\left(0, k_y v^R_{\omega\mathbf{k}_\perp}, -k_x v^R_{\omega\mathbf{k}_\perp}, 0\right), \qquad V^{R(2)}_{\omega\mathbf{k}_\perp\,b} = \frac{1}{k_\perp}\left(\partial_z v^R_{\omega\mathbf{k}_\perp}, 0, 0, \partial_t v^R_{\omega\mathbf{k}_\perp}\right). \tag{2}$$

We define left vector Rindler modes by simply replacing $R \to L$ in the above. Using these we can define ***vector Unruh modes***

$$W^{1(\kappa)}_{\omega\mathbf{k}_\perp b} := \frac{V^{R(\kappa)}_{\omega\mathbf{k}_\perp\,b} + e^{-\pi\omega/a}V^{L(\kappa)*}_{\omega\,-\mathbf{k}_\perp b}}{\sqrt{1-e^{-2\pi\omega/a}}}, \qquad W^{2(\kappa)}_{\omega\mathbf{k}_\perp b} := \frac{V^{L(\kappa)}_{\omega\mathbf{k}_\perp\,b} + e^{-\pi\omega/a}V^{R(\kappa)*}_{\omega\,-\mathbf{k}_\perp b}}{\sqrt{1-e^{-2\pi\omega/a}}}, \tag{3}$$

with $\kappa = 1, 2$; they are positive energy in all of Minkowski spacetime with regards to inertial time t.

Classical Unruh mode expansion

We use these vector Unruh modes to expand the retarded solution on the asymptotic future Σ_+

$$Rj_b(x) = -\sum_{\sigma=1}^{2}\sum_{\kappa=1}^{2}\int_0^\infty \mathrm{d}\omega \int_{\mathbb{R}^2}\mathrm{d}^2\mathbf{k}_\perp \left[\langle W^{\sigma(\kappa)}_{\omega\mathbf{k}_\perp}, Ej\rangle_{\mathrm{gKG}}\, W^{\sigma(\kappa)}_{\omega\mathbf{k}_\perp\,b}(x) + \text{c.c.}\right]. \tag{4}$$

Using our current (1), the non-zero amplitudes are given by

$$\langle W^{1(2)}_{\omega\mathbf{k}_\perp}, Ej\rangle_{\mathrm{gKG}} = -\frac{iq}{\pi^2}\sqrt{\frac{e^{\pi\omega/a}}{a}}K'_{i\omega/a}(a^{-1}k_\perp)\,\frac{\sin(\omega T)}{\omega} + \mathcal{I}^{1(2)}_{\omega\mathbf{k}_\perp}, \tag{5a}$$

$$\langle W^{2(2)}_{\omega\mathbf{k}_\perp}, Ej\rangle_{\mathrm{gKG}} = -\frac{iqe^{-\pi\omega/a}}{\pi^2}\sqrt{\frac{e^{\pi\omega/a}}{a}}K'_{i\omega/a}(a^{-1}k_\perp)\,\frac{\sin(\omega T)}{\omega} + \mathcal{I}^{2(2)}_{\omega\mathbf{k}_\perp}, \tag{5b}$$

where $\mathcal{I}^{\sigma(\kappa)}_{\omega\mathbf{k}_\perp}$ are the inertial movement contributions. Taking the limit $T \to \infty$ we can use the result $\omega^{-1}\sin(\omega T) \to \pi\delta(\omega)$ and $\mathcal{I}^{\sigma(\kappa)}_{\omega\mathbf{k}_\perp} \to 0$, then the retarded solution is

$$Rj_b(x) = -\frac{q}{2\pi}\sqrt{\frac{2}{a}}\int_{\mathbb{R}^2}\mathrm{d}^2\mathbf{k}_\perp\left[iW^{2(2)}_{0\mathbf{k}_\perp\,b}(x) - iW^{2(2)*}_{0\mathbf{k}_\perp\,b}(x)\right]K_1(a^{-1}k_\perp), \tag{6}$$

where its clear ***only zero-Rindler-energy modes contribute to the field***. If we compute these integrals on the $t > |z|$ region, we find the 4-potential originally reported by Born [11], up to a gauge transformation.

Quantum Unruh mode expansion

We can use the Unruh modes to expand the homogeneous part of the field in both the asymptotic past (Σ_-, out field) and the asymptotic future (Σ_+, in field), each with their own annihilation and creation operators and their respective vacua $\hat{a}^{\mathrm{in}}_{(j)}|0^M_{\mathrm{in}}\rangle = \hat{a}^{\mathrm{out}}_{(j')}|0^M_{\mathrm{out}}\rangle = 0$ [where (j) and (j') label an appropriate set of quantum numbers]. Using our Unruh modes for the out field, we connected these vacuum states using the S-matrix $|0^M_{\mathrm{in}}\rangle = \hat{S}|0^M_{\mathrm{out}}\rangle$, which, for the case $T \to \infty$ reads explicitly:

$$|0^M_{\mathrm{in}}\rangle = \bigotimes_{\mathbf{k}_\perp\in\mathbb{R}^2} e^{-q^2|K_1(a^{-1}k_\perp)|^2 T_{\mathrm{tot}}/(8a\pi^3)}\exp\left[\frac{iq}{2\pi}\sqrt{\frac{2}{a}}K_1(a^{-1}k_\perp)\,\hat{a}^\dagger_{\mathrm{out}}(W^{2(2)}_{0\mathbf{k}_\perp})\right]|0^M_{\mathrm{out}}\rangle, \tag{7}$$

i.e., ***a superposition of zero-Rindler-energy photons***. The S-matrix also allows us to see that the in vacuum is a multimode coherent state of the out operators

$$\hat{a}_{\mathrm{out}}(W^{\sigma(\kappa)*}_{\omega\mathbf{k}_\perp})|0^M_{\mathrm{in}}\rangle = \langle W^{\sigma(\kappa)}_{\omega\mathbf{k}_\perp}, Ej\rangle_{\mathrm{gKG}}|0^M_{\mathrm{in}}\rangle, \tag{8}$$

from where we see that the expectation values of the number operators per transverse momentum is

$$N(\mathbf{k}_\perp) := \sum_{\sigma=1}^{2}\sum_{\kappa=1}^{2}\int_0^\infty \mathrm{d}\omega\,\langle 0^M_{\mathrm{in}}|\left[\hat{a}^\dagger_{\mathrm{out}}(W^{\sigma(\kappa)}_{\omega\mathbf{k}_\perp})\hat{a}_{\mathrm{out}}(W^{\sigma(\kappa)*}_{\omega\mathbf{k}_\perp})\right]|0^M_{\mathrm{in}}\rangle = \frac{q^2}{4a\pi^2}\left|K_1(a^{-1}k_\perp)\right|^2 T_{\mathrm{tot}}, \tag{9}$$

when $T \to \infty$, which is consistent with the detector excitation rates reported by Higuchi et al. [6]. We also find the expectation values of the 4-potential, Faraday tensor and stress-energy tensor:

$$\langle 0^M_{\mathrm{in}}|\hat{A}^{\mathrm{out}}_a|0^M_{\mathrm{in}}\rangle = Ej(x) = -Rj(x), \tag{10a}$$

$$\langle 0^M_{\mathrm{in}}|\hat{F}^{\mathrm{out}}_{ab}|0^M_{\mathrm{in}}\rangle = \langle 0^M_{\mathrm{in}}|(\nabla_a\hat{A}^{\mathrm{out}}_b - \nabla_b\hat{A}^{\mathrm{out}}_a)|0^M_{\mathrm{in}}\rangle = -RF_{ab}, \tag{10b}$$

$$\langle 0^M_{\mathrm{in}}|:\hat{T}^{ab}_{\mathrm{out}}:|0^M_{\mathrm{in}}\rangle = g_{cd}RF^{ac}RF^{bd} - \frac{1}{4}g^{ab}RF^{cd}RF_{cd} = RT^{ab}, \tag{10c}$$

which coincide with their classical counterparts.

Summary of results

- Extended definition of Unruh modes to be vector valued.
- Used these to decompose the potential produced by a charge accelerated a finite amount of time.
- Showed that the field produced by an infinitely accelerated charge only depends on zero-Rindler-energy modes.
- Connected the descriptions in the asymptotic past and future.
- The future vacuum is a multimode coherent state.
- Observable quantities in the future correspond with the classical counterparts.
- Showed that the future vacuum of the field, when the charge is accelerated an infinite amount of time, is produced by a bath of past zero-Rindler-energy photons.

Forthcoming Research

We wish to expand this methodology to study gravitational radiation, to then analyse how the beginning state of the system influences the radiation content detected in the asymptotic future.

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